Numeracy Skills Revision

(Compiled by K. M. Pitchford, AST Mathematics, Woolston High School)

Index

Introduction

Inside this booklet you will find lots of useful (& free) information about using Numeracy skills.

What is Numeracy? I've never done Numeracy!
Well, actually you have done Numeracy – it just might not have been called that. Numeracy is the ability to use number related problem solving skills. Consider all the subjects you have done:

P.E.
Using a tape measure for long jump, timing the 100m

Food Technology
Measuring out ingredients, comparing nutritional values

Languages
Learning new ways to describe time, role play with currency

Technology
Using practical equipment to accurately measure

Science
Measuring out components for experiments, reading scales, interpreting results from tables and diagrams

Humanities
Using map scales, using timelines, explaining what data tables mean

Music
Composing pieces and accurately writing bars of music

Art
Scaling up/down, using ratios of colours to vary shades and tones

ICT
Knowing what calculations to perform in spreadsheets

Numeracy isn't just doing Maths, it is being confident in using numbers and number related skills. You just need to be able to apply what you know already.

This guide is designed to give you a helping hand with the basic skills you will need in all your GCSE subjects. So don't let it fester in the bottom of your bag, have a quick flick through and see what Numeracy could do for you.

Word Meanings

Remember that words have common meanings
A TRIcycle means three wheels, so a TRIangle has three corners.
A QUADbike has four wheels and a QUADrilateral has four sides.
A PARALLELogram has PARALLEL sides.

Trivia
Spot the odd one out: Bicycle, Binary, Bikini, Bisect, Biped

Answer:
Bikini – this doesn't mean two parts, it was named after Bikini Atoll the site of the American atomic bomb testing.

Pronunciation

This may seem obvious, but numbers such as 34.67 are often mispronounced. People read .67 as point sixty-seven, forgetting that it is actually six-tenths and 7 hundredths or just sixty-seven hundredths. The correct phrase would be thirty-four point six seven.

Big numbers are equally confusing – the best way to deal with them is to split them into chunks of three:

347890122 becomes 347 890 122

Each gap is a new word:
347 million 890 thousand 122

So you have three hundred and forty seven million, eight hundred and ninety thousand, one hundred and twenty-two!

…illions!

One million = one thousand thousands = 1 000 000

One billion = one thousand millions = 1 000 000 000

One trillion = one thousand billions 1 000 000 000 000

Calculators

General advice: Get your own scientific calculator!

This is really important because there are lots of different makes and models and they all work slightly differently: you need to know how to use yours.

- When you buy a calculator, don't throw away the instructions or draw over the help list in the lid. You never know when you'll need it.
- Yes- mobile phones have calculators, but they only do basic functions and are not to be used in school and are banned in exams.
- If you use a calculator to answer a question, make sure you note down what you did. This gives you something to revise from and also shows your teacher what you did, in case you got it wrong. You can also get method marks in exams.

Check your answer

- Is it reasonable? If the mean population of towns in Cheshire is 27 people, does this seem like a sensible result? It's very easy to hit the wrong key and get a silly answer.
- Substitute the result back into the problem, to check it works.
- Estimate the answer. Round off the numbers and use a non calculator technique. Is your answer similar to the calculator result?

Remember, a calculator isn't just for a Maths exam. You may need it in several exams – so just leave it in your pencil case for all your exams.

Equipment

The basic equipment for exams is:

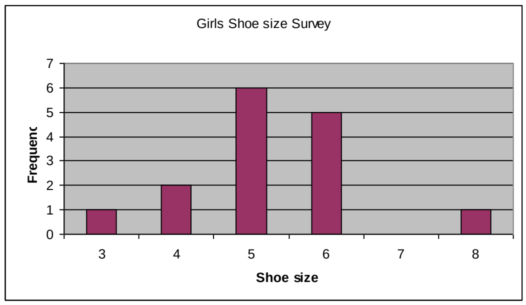
- Pencil (with eraser and sharpener if necessary)
- Ruler
- Pen (with either spare cartridges/refills or a spare pen)
- Compasses (make sure you have a pencil that fits in your compasses)
- Protractor (180° or 360° - you decide)
- A calculator (preferably scientific) – this is mainly for all Key Stage 3 students and also Year 6 students who will sit the level 6 SATs).

Observation vs. Interpretation

In basic terms, an observation is describing what can be seen in a diagram. It is collecting together facts.

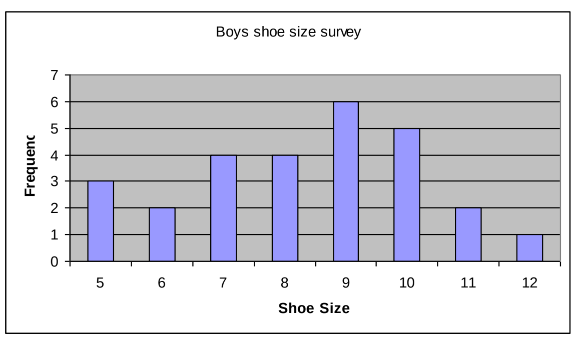
Interpretation is taking facts and explaining what they mean.

Example



Observation: Size 5 has the biggest bar.

Interpretation: Size 5 is the most popular answer, which means it is the modal shoe size (the most common).



Observation: There are more bars on the boys chart than the girls.

Interpretation: The range of the boys shoe sizes is greater than the girls. This means there is more variety in boys sizes. However they overlap, so some boys and girls can be the same size.

Vocabulary

Some common words have different meanings in Mathematics, compared with other subjects. Make sure you understand the correct vocabulary for each exam. The following are mathematical meanings:

Product – multiply
E.g. the product of 3 and 4 is 12.

Sum – add
E.g. the sum of 5 and 11 is 16.

Difference – subtract
E.g. the difference between 12 and 7 is 5.

Range – how spread out is the data
E.g. 2, 3, 6, 6, 8, 11, 13
The range of the data is from 2 to 13, which is 11.

Factor – will divide into exactly
E.g. 7 is a factor of 21.

Prime – only has two factors, 1 and itself
E.g. 2, 3, 5, 7, 11, 13 … are prime numbers.

Even – divides exactly by 2
E.g. 6 is even, but 9 isn't.

Significant (figures) – important values
E.g. 576306 rounded to 2 significant figures is 580000

Interpreting questions
Calculate means 'to work out' not use a calculator.
(A calculator used to be a person who did calculations for a living)

Evaluate means work out

Dictionary
If your biggest problem is working out what on earth a question is about, see if a mathematical dictionary helps.

Non-calculator techniques

Mental Strategies

Partitioning

45 + 37 = (5 + 7) + (40 + 30) = 12 + 70 = 82

67 – 32 = 67 – 30 – 2 = 37 – 2 = 35

Near doubles

45 + 46 = 45 × 2 + 1 = 90 + 1 = 91

Adjusting/Compensating

37 + 199 = 37+200 – 1 = 237 – 1 = 236

Multiples of 10

2005 – 1996 = 2005 – 2000 + 4 = 5 + 4 = 9

Counting on

87 – 55 =	55 + **5** = 60
	60 + **20** = 80
	80 + **7** = 87

So **5** + **20** + **7** = 20 + 12 = 32

Doubling

223 × 8 = 446 × 4 = 892 × 2 = 1784

Equivalent multiplication

35 × 18 = 35 × 2 × 9 = 70 × 9 = 630

Written methods

Expanded addition

432 + 326

Either

	400	30	2
+	300	20	6
	700	50	8

So 432 + 326 = 758.

Grid method

58 × 34

×	**50**	**8**	
30	1500	240	1740
4	200	32	+ 232
			1972

Division by chunking

Division by chunking involves subtracting easy 'chunks' of numbers away, for example a multiple of ten.

$2295 \div 85$

$$10 \times 85 = 850$$

(10) $2295 - 850 = 1445$

(10) $1445 - 850 = 595$ (can't minus anymore 850s)

When it becomes impossible to subtract anymore chunks of ten, move to a smaller number, for example a multiple of 5.

$$5 \times 85 = 425$$

(5) $595 - 425 = 170$ (can't minus anymore 425s)

The number being divided is considerably smaller, so the chunks are smaller. It is also possible to just repeatedly subtract by 85 at this stage.

$$2 \times 85 = 170$$

(2) $170 - 170 = 0$

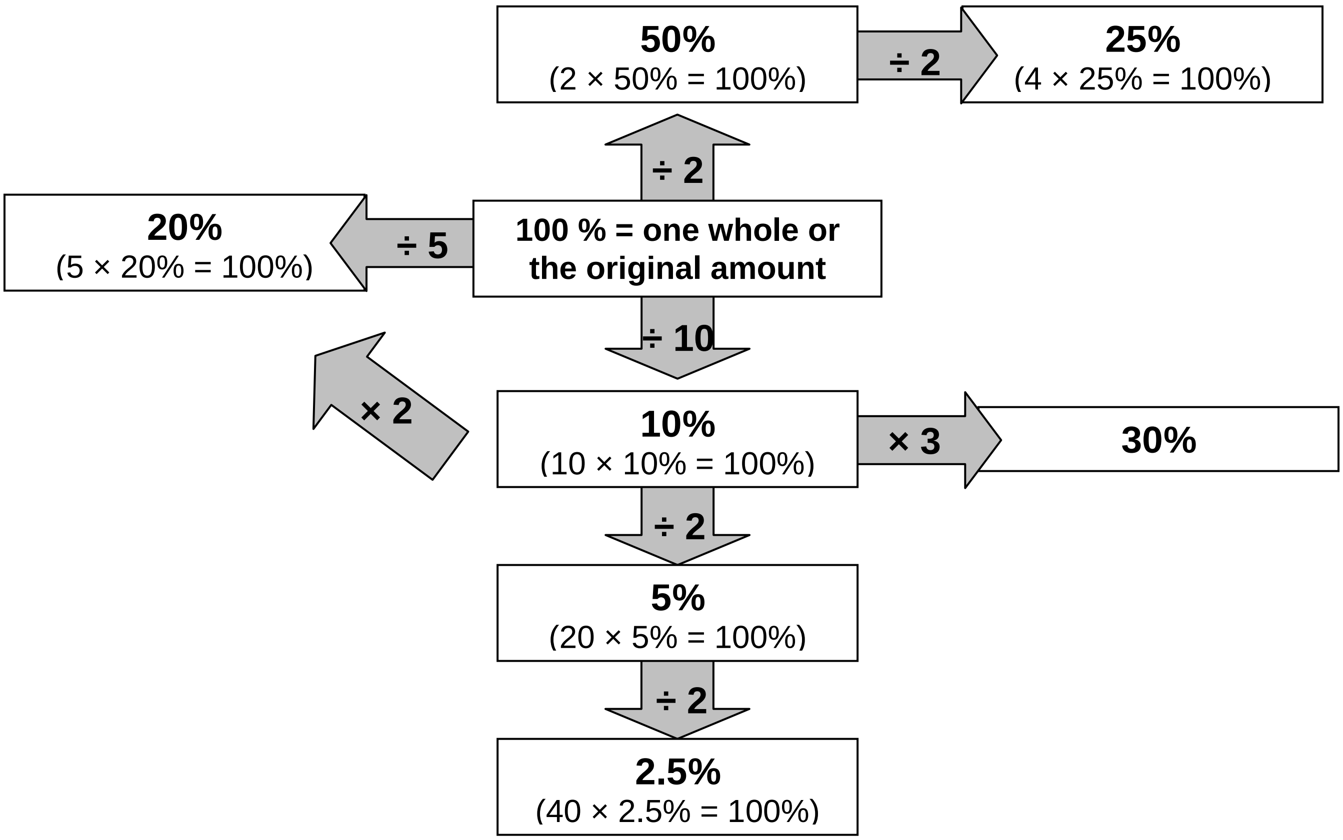
The tricky bit in this method is keeping track of what you have subtracted, hence the numbers in brackets.

Now $10 + 10 + 5 + 2 = 27$, so: $2295 \div 85 = 27$

There are many more non-calculator strategies. Try to find the best one for you.

Percentages: Non-calculator techniques

**Always find 10% first (the only one where you can divide by 10)



Simple percentages can be found by dividing by 10 (10%) or 2 (50%), followed by other basic operations.

Example

Find the VAT (17.5%) for a toaster costing £30.

Find 10%:	£30 ÷ 10 = £3
Find 5%:	£3 ÷ 2 = £1.50
Find 2.5%:	£1.50 ÷ 2 = £0.75
Find 17.5%:	£5.25

Percentages: Calculator techniques

Percent means ‘per hundred’ or ‘out of 100’.

To find 48% of 67kg, many people would divide by 100 to find 1% and then multiply by 48 to find 48%. Although this technique will give the correct answer it is not the most efficient use of a calculator.

Consider 78%:

$$78\% \quad = \quad \frac{78}{100} \quad = \quad 0.78$$

So to find 78% of an amount, just multiply by 0.78.

This method is very useful for percentage increase and decrease:

20% increase = original amount + 20% = 100% + 20% = 120% = 1.2

36% decrease = original amount - 36% = 100% - 36% = 64% = 0.64

Example: Increase 560 by 32%
Old method
1% of 560 is 5.60
So 32% is 5.6 × 32 = 179.2
Then 560 + 179.2 = <u>739.2</u>

New method
1.32 × 560 = <u>739.2</u>

Fractions

Fractions are not scary!

Don't believe it? Can you measure to the nearest eighth in inches? Can you tell the time on an analogue clock? Ever used a non-metric cookbook? Can you read music? If you answered 'yes' to any of those things then you already know a bit about fractions.

Vocabulary

$$\frac{\text{Numerator}}{\text{Denominator}}$$

Now this first hint may seem picky, but it's important that fractions are written down properly. The reasoning behind this is that subsequent calculations with fractions are less likely to have errors. A consistent approach ensures good practice.

4/5	or	$\frac{4}{5}$
No		Yes

Fractions of amounts

To find $\frac{4}{7}$ of £350

Find $\frac{1}{7}$: £350 ÷ 7 = £50 Then multiply by 4: £50 × 4 = £200

Multiplying fractions

This is the easy one:

$$\frac{2}{3} \times \frac{5}{11} = \frac{2 \times 5}{3 \times 11} = \frac{10}{33}$$

Just multiply the numerators & denominators.

Dividing fractions

This is nearly as easy as multiplying:

$$\frac{3}{5} \div \frac{2}{7} = \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{7}\right)} = \frac{3}{5} \times \frac{7}{2} = \frac{21}{10} = 2\ \frac{1}{10}$$

The technique is to invert the fraction you are dividing by and then multiply.
(invert means turn upside down in this context)

Equivalent fractions and simplifying

These two processes are directly related. To simplify, you are making a fraction with large numbers into an equivalent fraction, with smaller (easier to use) numbers.
Easy example

$$\frac{60}{120} = \frac{1}{2}$$

The technique is very straight forward and can be approached in one of two ways:

$$\frac{48}{72}$$

Look at the fraction – which multiplication table are the numbers both in?
48 and 72 are both in the Fours, so:

$$\frac{48}{72} = \frac{12 \times 4}{18 \times 4} = \frac{12}{18}$$

By cancelling out the 4s, the new fraction is $\frac{12}{18}$.
This process is repeated until the fraction can't be simplified anymore.

$$\frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3}$$

An alternative way to approach this is find a number that the will divide exactly into the numerator and denominator. Divide and write down the new fraction. Repeat this until you can't simplify anymore. Both methods are virtually identical – it just depends if you prefer multiplying or dividing.

Equivalent fractions can be found using the reverse process:

$$\frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35}{45}$$

Improper and mixed fractions
Sometimes you get a fraction where the numerator is bigger than the denominator – this is just like a division with a remainder.

$$\frac{50}{6} = 50 \div 6 = 8rem2 = 8\ \frac{2}{6} = 8\ \frac{1}{3}$$

The answer is a mixed fraction – a mixture of whole number and fraction.

To convert from a mixed fraction to an improper fraction, reverse this process.

$$5\ \frac{2}{7} = \frac{5 \times 7 + 2}{7} = \frac{35 + 2}{7} = \frac{37}{7}$$

Addition and Subtraction

The key facts to remember when adding or subtracting fractions are:

- The denominator tells you the size of the fraction
- The numerator tells you how many of that type of fraction you have

So

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$ This is okay as the problem involves thirds

When you get two different denominators, you need to find a new denominator that they have in common – the Common Denominator.

$$\frac{2}{5} + \frac{4}{7}$$

In this case, 5 and 7 both go into 35. So change the fractions to equivalent fractions with the denominator 35.

$$\frac{2}{5} = \frac{14}{35} \qquad \frac{4}{7} = \frac{20}{35}$$

This means that:

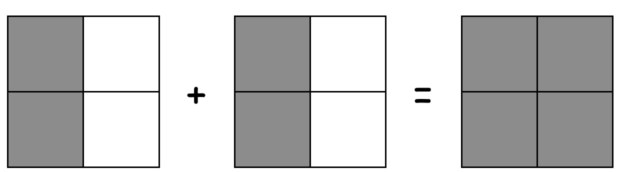
$$\frac{14}{35} + \frac{20}{35} = \frac{14+20}{35} = \frac{34}{35}$$

The Big Question: Why don't you add the denominators?

Think about this simple problem:

What is a half plus a half? Easy – one whole

Visually:



If you do this calculation and add the denominators you get:

$$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4}$$

Hang on – two quarters is the same as a half. How can a half plus a half make a half? That makes no sense.

Remember, the denominators describe how big the fraction is.

Order of Operations: BIDMAS

B rackets — **Start**

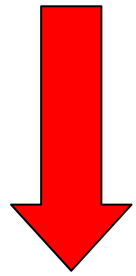
I ndices

D ivision

M ultiplication

A ddition

S ubtraction — **Finish**



Problem

- Pick a number, for example 5.
- Then + 3, × 2
- Check your result on a calculator.

The answer is 16 isn't it? Or is it 11?
The calculation was: 5 + 3 × 2

$5 + 3 \times 2 = 8 \times 2 = 16$

$5 + 3 \times 2 = 5 + 6 = 11$

Confusing isn't it?
To make it worse simple calculators work out problems as you go, giving the answer 16, and scientific calculators follow algebraic logic, giving the answer 11!

The 'correct' answer is 11, because you use the order of operations: multiplication happens before addition. BIDMAS is used as a quick way of remembering the correct order.

Spreadsheets probably use BIDMAS most in 'real-life'.

Aside: What on earth are Indices?

Index Notation is :

The notation in which a product such as a×a×a×a is recorded as a^4. In the example the number 4 is the index (plural: indices).

So, this means:
$7^3 = 7 \times 7 \times 7 = 343$ not to be confused with $7 \times 3 = 21$

Calculator
Scientific calculators have a button labelled x^y or y^x or ^. They make it very quick to workout indices.
If you had the question 'Find the value of 2^8?' which would you rather do on a calculator?

Method A
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = ?$

Method B
2 x^y 8 = ?

The important bit: Presentation
Does this mean 3×4 or 34 or 3^4? 34

Much better would be: 3^4

Which of these mean 2×n and which mean n^2?

2n n2 n^2 n2

Clear writing makes life less confusing. Remember:
$2n = 2 \times n = n + n$
$n^2 = n \times n$

Interesting Fact
Anything with an index number of 0 equals 1.
E.g. $x^0 = 1$ $569^0 = 1$ $(-5)^0 = 1$ $670{,}003{,}629^0 = 1$

Averages

There are three types of average: Mean, Median & Mode

Mode Mode Mode Mode Mode Mode

The Mode is the answer that occurs the most.
It is the most popular (common) answer.

↓
MEDIAN

The Median is the middle value when all the numbers are in order.
E.g.
3,6,8,2,6,0,1,7,5 → 0,1,2,3,4,6,6,7,8
4 is the middle number, so the median is 4.

2,6,4,8,9,2,4,6 → 2,2,4,4,6,6,8,9
4 and 6 are the middle numbers, so the middle of 4 and 6 is 5. The median is 5.

$$\frac{M + E + A + N}{4}$$

The mean is when the values are shared out equally. This means you total them up and divide by how many values there were.
E.g.
0, 7, 9, 3, 1
The total is 20 and there are 5 values, so:
Mean = 20 ÷ 5 = 4

Drawing graphs

Essential: A sharp pencil and ruler

- The across axis is x (Get it? X is a cross)
- The vertical axis is y (Y is taller than x)
- Co-ordinates are (x, y), which is alphabetical order.
- Mark points with a small × because dots can be messy and inaccurate.

- Scales on axes are equally spaced, just like a number line.
- You always label the line, not the space (unless it's a bar chart).
- Don't forget to put titles on each axis and an overall title on your diagram.

- The overall accuracy of your diagram is more important than pretty colouring in.
- If you are using colours to show different information, use pencil first then go over in colour. Don't forget a key to explain what is going on.

Pie charts

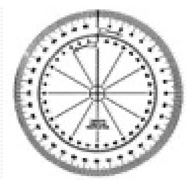
Essential: a sharp pencil, a ruler, a protractor, a pair of compasses
Optional: a calculator

There are two different techniques when it comes to drawing pie charts:

- Raw Data – a list of quantities
- Percentages – a list of percentages

So the first decision you make is which method is best for the data you have.

Pie chart Scales



360° protractor

A pie chart scale looks a bit like a 360° protractor. It's a circle divided up into 100 parts, usually labelled every 5% or 10%.

Advantages

- If you have a list of percentages and need to draw a pie chart, a pie chart scale is very handy.
- Some people find them less confusing than protractors.

Disadvantages

- What if you don't have a list of percentages? You'll have to change all your values into percentages first. Can you remember how to do that?
- If you rely on a pie chart scale, you might forget the other methods for drawing a pie chart. This could cause problems.

Overall, it's up to you to find a method you are happy with.

Pie charts: Raw data

This table represents a survey of teenagers and which brand of mobile phone they own.

Mobile Phone Brand	Nokia	Samsung	Sony Ericsson	Motorola	LG	Other
Frequency	59	29	34	32	16	10

First of all, add up how many pieces of information you have:

59 + 29 + 34 + 32 + 16 + 16 + 10 = 180

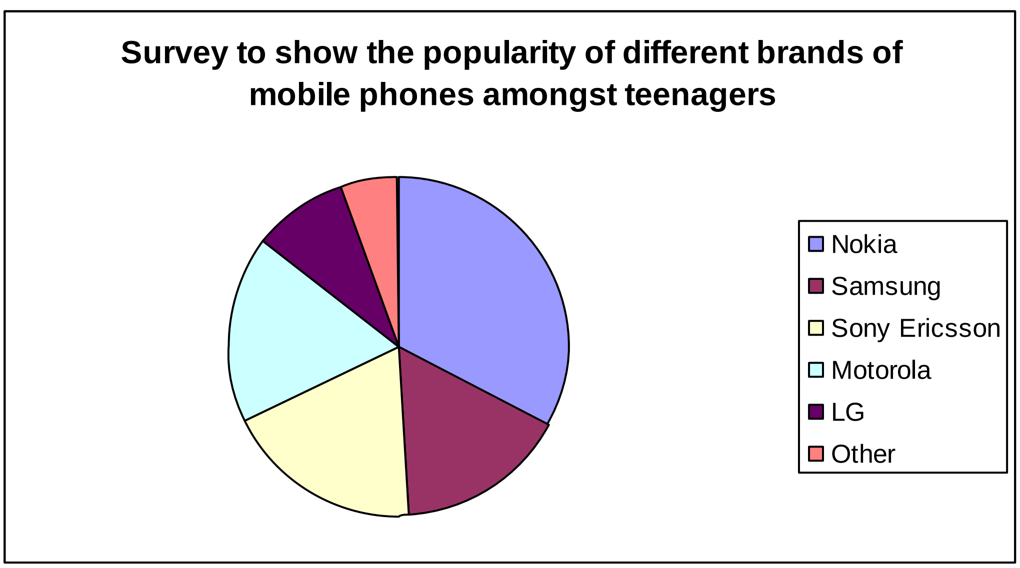
Then calculate how many degrees represent one person:

$360° \div$ (total number of people) $= 360° \div 180 = 2°$

So each person is represented by 2°. This means that if 59 people own a Nokia phone, then Nokia is $59 \times 2° = 118°$.
It's quite useful to add an extra line to your table for working out.

Mobile Phone Brand	Nokia	Samsung	Sony Ericsson	Motorola	LG	Other
Frequency	59	29	34	32	16	10
Calculations	59x2°=118°	29x2°=58°	34x2°=68°	32x2°=64°	16x2°=32°	10x2°=20°

Lastly, before you start to draw out these angles, check they add up to 360°.



Pie charts: Percentages

This table represents a survey of DVD rentals

DVD Genre	Comedy	Thriller	Sci-Fi	Action	Horror	Other
Percent (%)	21	19	15	24	13	8

Since the table contains percentages, they should add up to 100% - it's not a bad idea to check this.

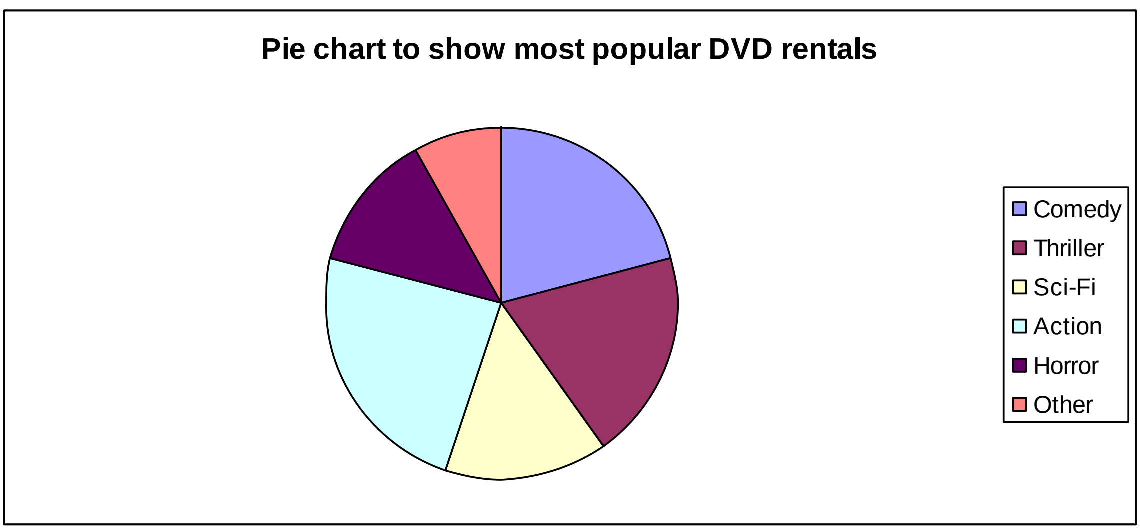
Then calculate how many degrees represent one percent: 360 ÷ 100 = 3.6

So every time you use percentages with a pie chart, 1% is 3.6°.

Now calculate the angles, by multiplying each value by 3.6°
As before, it's quite useful to add an extra line to your table for working out.

DVD Genre	Comedy	Thriller	Sci-Fi	Action	Horror	Other
Percent (%)	21	19	15	24	13	8
Calculations	21x3.6°=75.6°	19x3.6°=68.4	15x3.6°=54°	24x3.6°=86.4°	13x3.6°=46.8°	8x3.6°=28.8°

Lastly, before you start to draw out these angles, check they add up to 360°.



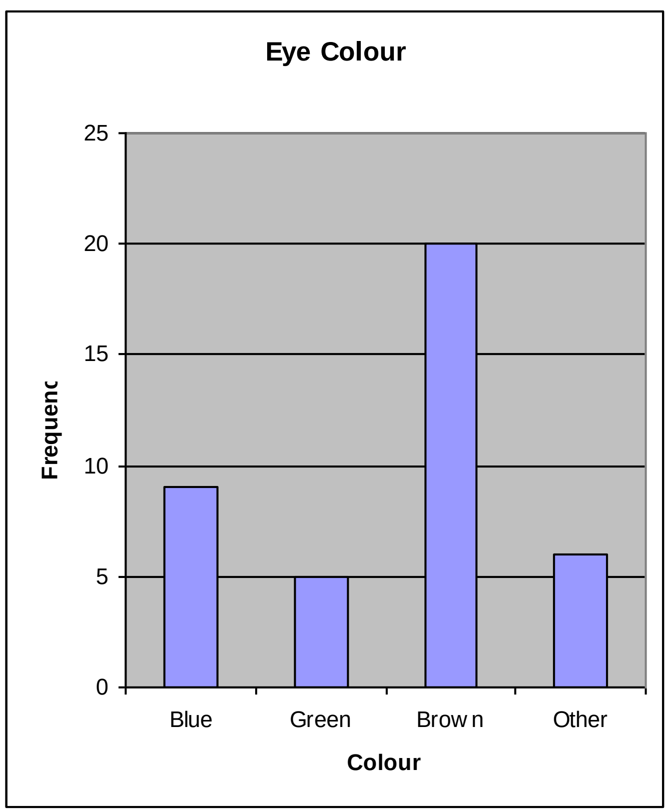
Bar charts vs. Histogram

To understand when to use a bar chart and when to use a histogram it is important to know what the differences are between them:

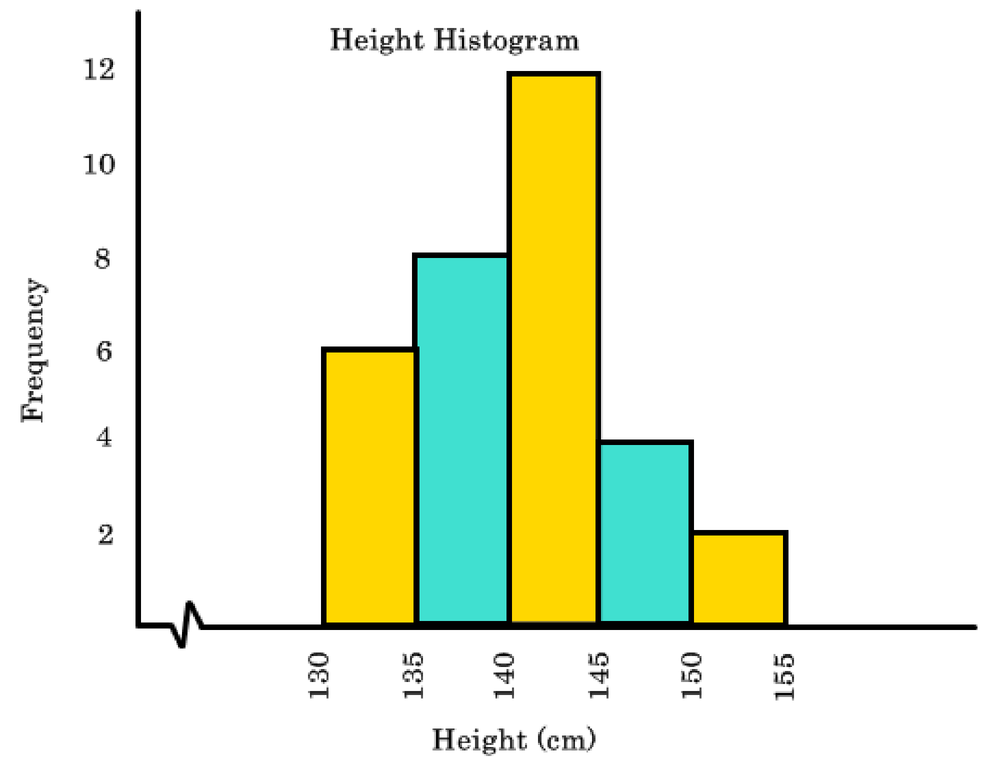
	Bar chart	**Histogram**
Display type	Rectangular bars	Rectangular bars
Data type	Discrete, not necessarily ordered numerically e.g. eye colour, shoe size	Continuous measured data e.g. height
Axis Labelling	On the axis the middle of each bar is labelled.	On the axis the ends of each bar are labelled – it reads like a normal equally spaced axis.
Spacing	Bars are separate	Bars are touching
Frequency Polygon	No	Yes – join the midpoints at the top of each bar.

Examples

Bar chart



Histogram



Summary

- Bar charts are used to visually represent data.
- Histograms give a visual representation and can also be used for higher order mathematical analysis. Drawing a frequency polygon on a histogram allows trends and patterns to be investigated.

Additional Note 

This symbol may look like a blip from a cardiac readout, but it actually means the scale on the axis has been condensed – a bit like folding up that part of the scale. It is good practice to only use it at the start of axes.

If you look at the histogram above, there were no values below 130. By using this device, there isn't a huge empty bit of histogram. It also allows the histogram to be drawn to an appropriate scale – this is especially important if you are using graph paper.

Scatter Graphs and Correlation

A scatter graph is used to compare two pieces of numerical information. The basic requirement is a list of pairs of numbers.

Drawing a scatter graph involves all of the skills needed to create the axes of a histogram (Refer to page 26) and use of co-ordinates (Refer to page 36). One key fact to remember is the points aren't joined up. If you try it, you end up with a bizarre dot-to-dot picture.

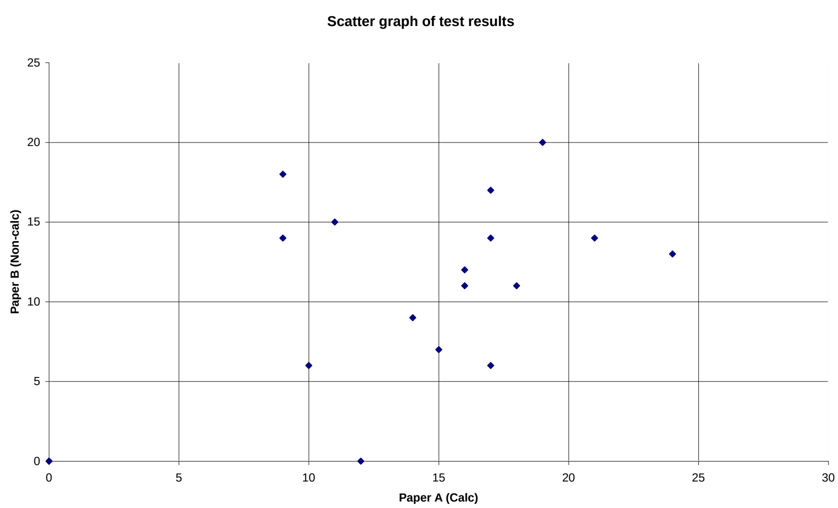
'Real – Life' Example

Height/Weight graphs are a common site in Healthcare offices, publications and websites. These will have been derived from research into national trends and medical advice. In actuality, they are an enhanced form of scatter graph.

Educational Example

A useful application is comparing exam performance on two different papers. It is easy to see if the same pupils did well (or not) on both papers without having to trawl through set lists.

Paper A	Paper B
32	**32**
11	15
9	18
17	6
19	20
10	6
21	14
17	17
14	9
15	7
17	14
0	0
16	11
9	14
18	11
12	0
16	12
24	13
0	0



Analysis

As you can see, the general trend is that pupils who performed well in Paper A also achieved a similar result in Paper B. This isn't a strong link, since several pupils weren't consistent. These trends are called *correlations*.

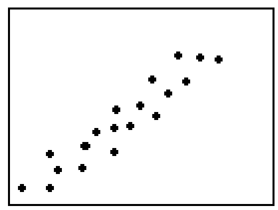
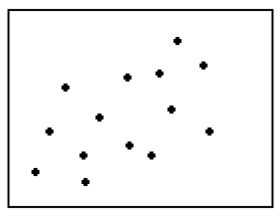
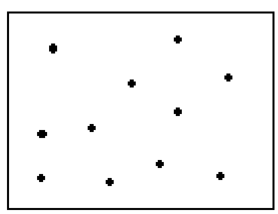
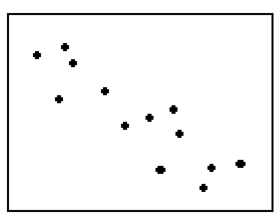
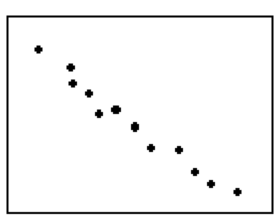
Correlation

Correlation means connection or relationship, so in a scatter graph it describes the link between the two sets of data. A strong correlation defines a clear trend, whereas a weak correlation is a vaguer trend or weak connection.

Types of trend

A *positive correlation* goes in a diagonal pattern from bottom left towards top right.

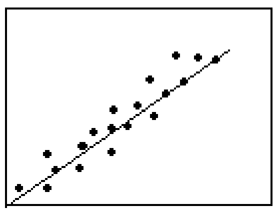
A *negative correlation* goes in a diagonal pattern from top left towards bottom right.

Correlation	Diagram	Example
Strong Positive		Distance travelled against petrol used
Weak Positive		Ice creams sold against daily temperature
No correlation		House number against salary
Negative		Age against car value for standard cars
Strong Negative		Disposable income against number of children

Line of best fit

The line of best fit is basically a line which best describes the trend on a scatter graph or set of experimental results. Lines can straight or a curve – Science is the subject where you are most likely to need a curved line of best fit.

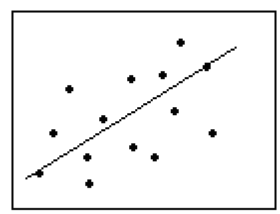
Common mistakes
The worst and most common mistake is joining the last point to the origin. This indicates a complete misunderstanding of what the line is meant to represent.



There are ten points above the line and four below it. The line is too low.

Negative correlations can be done equally badly.

Handy Hints: Straight line



- The line does not have to go through zero
- It must be drawn with a ruler
- The line should go through as many points at possible
- There should be roughly the same number of points above and below the line
- The line should extend to the edge of the graph area
- It should follow the trend of the graph
- There is very rarely a perfect solution – two people can draw different lines and still be correct

Tip: Hold the ruler on its edge on the graph and look down onto the paper. Move the ruler to find the best position and then carefully lie the ruler down and draw the line.

Handy Hints: Curved Line

- Follow the same rules as for a straight line, but don't use a ruler.
- Try to draw a smooth curve – try drawing lightly in pencil first.
- Make sure it doesn't look like a crazy dot-to-dot puzzle

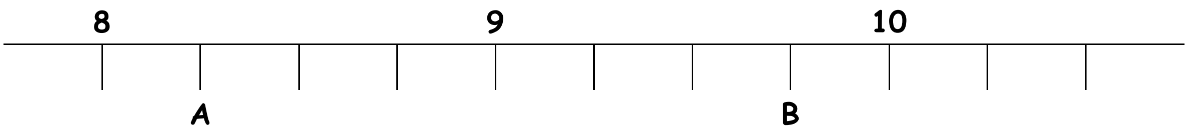
Reading Scales

Common Errors

The biggest mistake people make when using numerical scales is forgetting to check whether it is set to zero e.g. the tape measure isn't at the start. This sounds so obvious, but it really does happen.

The second problem is the ability to interpret unlabelled markings.

Example



Point A is read as 8.1 by those who think it is one space along or as 8.2 by those who realise it's a bigger space, but cannot quite figure out what. Similarly Point B is read as 9.8 or 9.9.

Technique

The correct method is to take everything one step at a time.

Firstly, how big is each gap? ***9 – 8 = 1***

Then, how many spaces are in each gap? ***4***

How much is each space worth? ***1 ÷ 4 = 0.25***

Finally use this information to read the scale.

Point A is 8 + 1 × 0.25 = 8.25

Point B is 9 + 3 × 0.25 = 9.75

Always remember to check your answer is reasonable.

Unit Conversion

The most common unit conversions used each day are weights and measures. This includes both metric and imperial units.

Metric

Length	Weight
1 km = 1000 m	1 tonne = 1000 kg
1 m = 100 cm = 1000 mm	1 kg = 1000 g
1 cm = 10 mm	1 g = 1000 mg

Imperial

Length	Weight
1 ft = 12 inches	1 lb = 16 oz
1 yd = 3 ft	1 st = 14 lb
1 mile = 5280 ft	1 ton = 2240 lb
1 mile = 1760 ft	

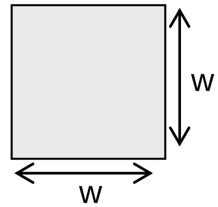
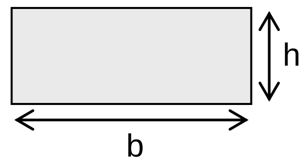
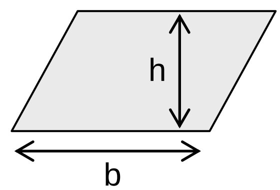
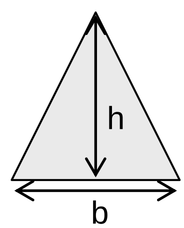
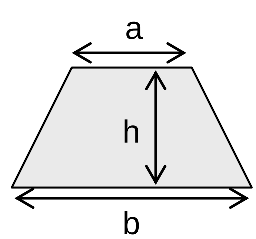
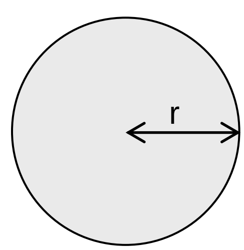
Common Conversions

5 miles = 8 km	miles to km	× 1.6
	km to miles	÷ 1.6
1 kg = 2.2 lb	kg to lb	× 2.2
	lb to kg	÷ 2.2
1 inch = 2.5 cm	inch to cm	× 2.5
	cm to inch	÷ 2.5

Handy Hint: How to cheat in any exam
A standard long ruler is 30cm – this is a strange number to pick until you remember we used to use the Imperial system. Standard rulers used to be one foot long – actually they still are. So if you take a standard ruler into an exam you can have an easy advantage.

Cheat 1: 12 inches is 1 foot
Cheat 2: 30 cm is 1 foot
Cheat 3: Compare the scales; you'll see that 1 inch = 2.5 cm
Cheat 4: If your ruler has millimetres, you can see that 1 cm = 10 mm

Areas of common shapes

Square Area = length × length = length^2 $A = w \times w = w^2$	
Rectangle Area = base × height $A = b \times h$	
Parallelogram Area = base × perpendicular height $A = b \times h$	
Triangle Area = (base × perpendicular height) ÷ two $A = b \times h \div 2$	
Trapezium Area = sum of the parallel sides × perpendicular height ÷ two $A = (a + b) \times h \div 2$	
Circle Area = pi × radius × radius $A = \pi \times r \times r = \pi r^2$ Circumference (perimeter) = $2\pi r$ *Note:* *- π can be approximated to 3.14* *- Do not square π when working out the area*	

Co-ordinates

Co-ordinates are used to describe where points are on a grid. There are different types of co-ordinates, but the most common type is Cartesian.

Essentials: Squared paper or a grid, a sharp pencil

X and Y

There are many ways to remember which axis is which, such as 'you must crawl, before you walk'. The simplest way is:

- The across axis is x (Get it? X is a cross)
- The vertical axis is y (Y is taller than x)

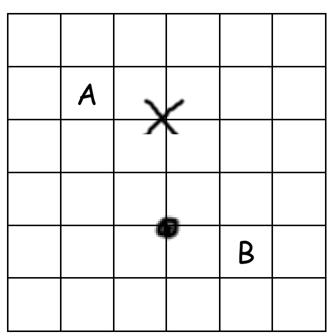
Scales on axes are equally spaced, just like a number line.
You always label the line, not the space (unless it's a bar chart).
The axes cross at 0, which is the point (0, 0). This is called the Origin.

Notation

- Co-ordinates are (x, y), which is alphabetical order.

Blobs and crosses

Which point on the grid is easier to see?

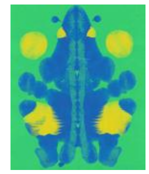


The centre of the cross is much easier to see, literally 'X marks the spot'.
Mark points with a small × because dots can be messy and inaccurate. This is especially true when using graph paper or trying to read small scales.

Symmetry

There are two forms of symmetry and everybody has met them, whether they realise it or not.

Line symmetry



Did you ever splodge paint on one half of a piece of paper, fold it over, press down and then unfold it to reveal a 'butterfly'? If you did that was your introduction to line symmetry.

Line symmetry is basically how many times you can fold a shape in half perfectly. Lines of symmetry are also referred to as mirror lines, for a similar reason – where can you put a mirror and not change the picture.

Symmetry is key to so many things – if you don't hold your body symmetrically whilst bouncing on a trampoline, you will most likely fall off. If you don't put up shelf supports symmetrically either the shelf (or the contents will fall off) or it will just annoy you that it doesn't look quite right.

Examples

- A square has four lines of symmetry, a rhombus has two
- Correctly drawn, the letters W, E, T, Y, U, A, D, K, C, V, B and M have one line of symmetry.
- VW, Toyota, Citroen and Mazda are just a few of the car logos with one line of symmetry.

Experiment

Find a photograph where you are looking straight at the camera. Carefully place a mirror down your nose. Look in the mirror. Is that really you? Research has shown that people find imperfections more attractive than totally symmetrically faces – in fact perfectly symmetrical faces look alien. So if your nose is crooked or one eyebrow is higher than the other, don't book the plastic surgeon – you might be messing up your chances of pulling the opposite sex!

Rotational symmetry

Every toddler who has had a shape sorter knows about rotational symmetry. If you have a triangular peg, it will fit into a triangular hole in three different ways. A square peg fits four ways. Of course, a circular peg fits in an infinite number of ways.

The mathematical description of the order of rotational symmetry would be how many times a shape fits exactly upon itself in a full (360˚) turn. Rotational symmetry isn't about how many sides a shape has; it's whether it fits on itself. Regular shapes (ones which have equal sides and angles) have rotational symmetry.

Examples

- A square has order four, but a rectangle has order two
- Correctly drawn, the letters Z, N, S, I, X and O have order two
- Both the Mercedes and Mitsubishi car badges have order three

Common Formulae

Some common formulae stare us in the face and we don't even know it.

Examples

What is speed measured in? MPH

There's the clue 'miles per hour' or 'miles / hour'.

So speed = miles ÷ hour = distance ÷ time.

Similarly, density can be measured in g/cm^3. This equates to grams ÷ cm^3, which implies the formula 'density = mass ÷ volume'.

Temperature

A more useful formula is the Centigrade to Fahrenheit conversion.

Fahrenheit to Celsius

$$C = 5 \times (F - 32) \div 9$$

And in reverse, Celsius to Fahrenheit

$$F = (9 \times C \div 5) + 32$$

Currency Conversion

There are many currencies around the world and the exchange rates change daily. This makes it useless to try and give a specific formula for conversion. However, a more general rule is:

GBP = Amount of currency ÷ (rate of £1 = ?)

GBP = Amount of currency × (rate of 1 unit = £?)

And to reverse this process, just work backwards

Amount of currency = GBP × (rate of £1 = ?)

Amount of currency = GBP ÷ (rate of 1 unit = £?)

Substitution into formulae

This sounds complicated, but it's just like a sporting match. If you substitute a player, they are replaced by someone to play the same position wearing a different number. Formulae are no different – if g = 5, then replace all g's with the value 5.

Common misconception

If y = 7, 6y + 1 = 67 + 1 = 68. In reality, 6y + 1 = 6 × 7 + 1 = 43

The reason is 6y means 6 × y, the multiplication sign has been dropped; if you think about it this makes sense. If you have 6 lemons, you wouldn't say 'I have lemons multiplied by six'. In the English language, we often take shortcuts and algebra is no different. Algebra could be considered the height of laziness as everything is abbreviated.

Rearranging formulae

Some revision guides recommend the triangle method for rearranging formulae. This involves writing a formula in a triangle and covering up the part you are trying to find

e.g.

This represents Speed, Distance and Time.
If you want to find T, cover it up.
This leaves D over S, which is D ÷ T

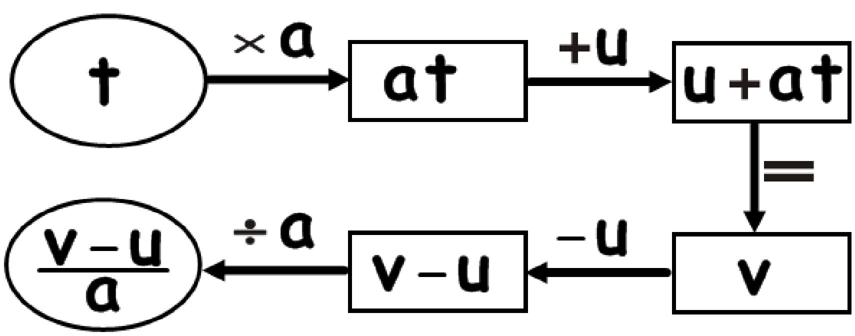
So long as the triangle is remembered correctly it will work, but doesn't promote understanding.

Techniques

The easiest way to rearrange a simple formula is to build it up and then reverse the process. Flowcharts are the quickest way, without getting 'bogged down' in algebra.

Example: Rearrange v = u + at, to find t.

This is the formula for final velocity, given start velocity, acceleration and time. The question asks for t (time), so build up the formula from there:



This gives the solution:

$$t = (v - u) \div a$$

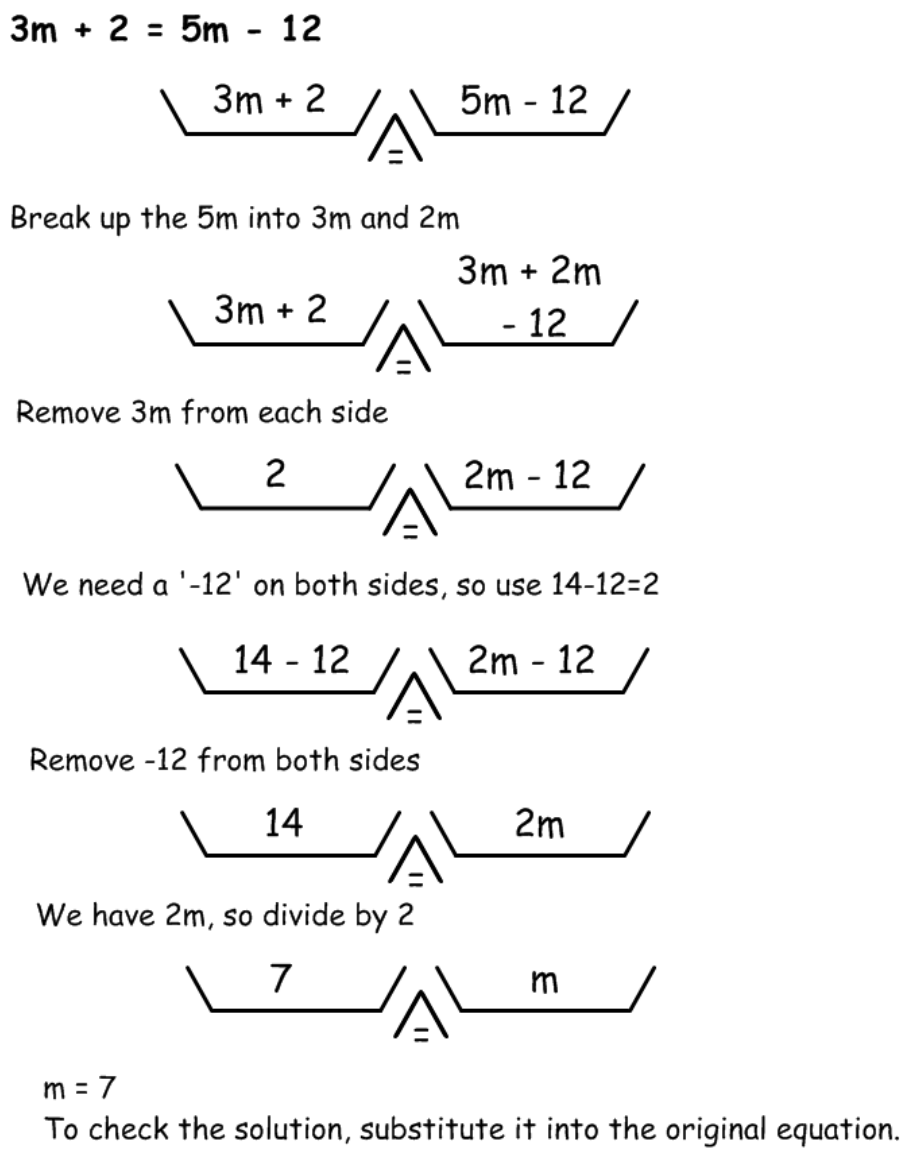
If you follow the flow chart through, you can see the formula is built up until it is complete and then reversed. The useful thing is that this technique will work for all basic equations (i.e. it won't work for $y = 2x^2 + 3x - 2$).

It is also possible to use straight forward algebraic manipulation.

Using 'scales' to solve equations

The idea behind using scales to solve equations is the concept of 'equals'. On traditional scales, the pans balance when they contain the same amount. An equation is only true if the left side of the equals sign remains the same as the right side of the equals sign.

Key idea: Whatever you do to one side you must do to the other.



More traditional techniques are available, but this visual method shows why everything must be balanced.